

# Study Guide

## Quadratic Formula 02/29/2012

### Quadratic Formula

A quadratic equation is a polynomial equation in which the highest power of the unknown variable is two.

An example of a quadratic equation is below.

$$x^2 + 6x - 91 = 0.$$

The format of a quadratic equation is  $ax^2 + bx + c = 0$ . Quadratic equations can be solved by factoring, graphing, or by using the quadratic formula. The quadratic formula is as follows:

Quadratic Formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

It can be found in any algebra textbook. This formula should be memorized.

To apply the formula to a quadratic equation, use the quadratic equation format given above as a guideline.

**Example 1:** Solve the quadratic equation.

$$\begin{array}{ccc} x^2 + 6x - 91 = 0 & & \\ \text{(1)} & \text{(2)} & \text{(3)} \\ x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-91)}}{2(1)} & x = \frac{-6 \pm \sqrt{400}}{2} & x = \frac{-6 \pm 20}{2} \\ \text{(4)} & & \\ x = \frac{-6 + 20}{2} \text{ and } x = \frac{-6 - 20}{2} & & \\ x = \frac{14}{2} & x = \frac{-26}{2} & \\ x = 7 \text{ and } x = -13 & & \end{array}$$

**Step 1:** Determine the values of a, b, and c and substitute them into the quadratic formula. a = 1, b = 6, and c = -91

**Step 2:** Determine the value under the radical symbol. 6 squared is 36 and -91 times -4 equals 364.  $36 + 364 = 400$

**Step 3:** The square root of 400 is 20 ( $20 \times 20 = 400$ ).

**Step 4:** Split the remaining problem into two problems:  $(-6 + 20) \div 2$  and  $(-6 - 20) \div 2$  and solve the two problems.

The answers are  $x = 7$  and  $x = -13$ .

**Example 2:** Solve the quadratic equation.

$$5x^2 + 2x + 8 = 4x^2 - 2x + 4$$

$$(1) 5x^2 + 2x + 8 = 4x^2 - 2x + 4$$

$$(2) 5x^2 + 2x + 8 = 4x^2 - 2x + 4$$

$$\begin{array}{r} -4x^2 \\ \hline x^2 + 2x + 8 = -2x + 4 \\ +2x \quad +2x \\ \hline \end{array}$$

$$x^2 + 4x + 8 = 4$$

$$\begin{array}{r} -4 \quad -4 \\ \hline x^2 + 4x + 4 = 0 \end{array}$$

$$(3) a = 1, b = 4, c = 4$$

$$(4) x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(4)}}{2(1)}$$

$$(5) x = \frac{-4 \pm \sqrt{16 - 16}}{2} = \frac{-4 \pm 0}{2}$$

$$(6) x = \frac{-4}{2} = -2$$

**Step 1:** Write the equation.

Subtract  $4x^2$  from both sides of the equation.

Then, add  $2x$  to both sides of the equation.

Finally, subtract 4 from both sides of the equation.

**Step 2:** This will put the equation in standard form.

**Step 3:** Determine the values of a, b, and c.

**Step 4:** Substitute the values of a, b, and c into the quadratic formula.

**Step 5:** Determine the value under the radical sign. The square root of 0 is 0.

**Step 6:** Solve for x.

Answer:  $x = -2$

The **discriminant** is the portion of the quadratic equation under the radical sign  $b^2 - 4ac$ . The discriminant properties below will give you vital information about quadratic equations.

1. If the discriminant is a perfect square, then the quadratic equation can be factored.
2. If the discriminant is greater than 0, then the equation has two real solutions.
3. If the discriminant is less than 0, then the equation has no real solutions.
4. If the discriminant is equal to 0, then the equation has one real solution.

**Example 3:** How many solutions does the following quadratic equation have?

$$3x^2 + 5x - 12 = 0$$

$$(1) a = 3, b = 5, c = -12$$

$$(2) (5)^2 - 4(3)(-12)$$

$$(3) 25 + 144 = 169$$

**Step 1:** Determine the values of a, b, and c.

**Step 2:** Substitute the values for a, b, and c into  $b^2 - 4ac$ . **Step 3:** Simplify the discriminant.

Since the discriminant is greater than zero, there are two real solutions.

If the discriminant is a perfect square, then the solutions are rational.

If the discriminant is not a perfect square, then the solutions are irrational.